

Multiple D2-Brane Action from M2-Branes

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Abstract

We study the detail derivation of the multiple D2-brane effective action from multiple M2-branes in the Bagger-Lambert-Gustavsson (BLG) theory and the Aharony-Bergman-Jafferis-Maldacena (ABJM) theory by employing the novel Higgs mechanism. We show explicitly that the high-order F^3 and F^4 terms are commutator terms, and conjecture that all the high-order terms are commutator terms. Because the commutator terms can be treated as the covariant derivative terms, these high-order terms do not contribute to the multiple D2-brane effective action. Inspired by the derivation of a single D2-brane from a M2-brane, we consider the curved M2-branes and introduce an auxiliary field. Integrating out the auxiliary field, we indeed obtain the correct high-order F^4 terms in the D2-brane effective action from the BLG theory and the ABJM theory with $SU(2) \times SU(2)$ gauge symmetry, but we can not obtain the correct high-order F^4 terms from the ABJM theory with $U(N) \times U(N)$ and $SU(N) \times SU(N)$ gauge symmetries for $N > 2$. We also briefly comment on the (gauged) BF membrane theory.

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I. INTRODUCTION

Inspired by the ideas that the Chern-Simons gauge theories without Yang-Mills kinetic terms may be used to describe $\mathcal{N} = 8$ superconformal M2-brane world-volume theory [1, 2], Bagger and Lambert [3, 4, 5], as well as Gustavsson (BLG) [6, 7] have successfully constructed three-dimensional $\mathcal{N} = 8$ superconformal Chern-Simons gauge theory with manifest $SO(8)$ R-symmetry based on three algebra. And then there is intensive research on the world-volume action of multiple coincident M2-branes [8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85]. Although the BLG theory is expected to describe any number of M2-branes, there is one and only one known example with gauge group $SO(4)$ for the positive definite metric [16, 20, 21]. At the level one of the Chern-Simons gauge theory, the BLG $SO(4)$ gauge theory describes two M2-branes on a R^8/Z_2 orbifold [13, 14]. Thus, it is very important to generalize the BLG theory so that it can describe an arbitrary number of M2-branes.

By relaxing the requirement of the positive definite metric on three algebra, three groups [24, 25, 26] proposed the so called BF membrane theory with arbitrary semi-simple Lie groups. However, the BF membrane theory has ghost fields and then the unitarity problem in the classical theory due to the Lorentzian three algebra. To solve these problems, the global shift symmetries for the bosonic and fermionic ghost fields with wrong-sign kinetic terms are gauged, which ensures the absence of the negative norm states in the physical Hilbert space [40, 43]. However, this gauged BF membrane theory might be equivalent to three-dimensional $\mathcal{N} = 8$ supersymmetric Yang-Mills theory [47] via a duality transformation due to de Wit, Nicolai and Samtleben [86].

Very recently, Aharony, Bergman, Jafferis and Maldacena (ABJM) have constructed three-dimensional Chern-Simons theories with gauge groups $U(N) \times U(N)$ and $SU(N) \times SU(N)$ which have explicit $\mathcal{N} = 6$ superconformal symmetry [44] (For Chern-Simons gauge theories with $\mathcal{N} = 3$ and 4 supersymmetries, see Refs. [87, 88]). Using brane constructions they argued that the $U(N) \times U(N)$ theory at Chern-Simons level k describes the low-energy limit of N M2-branes on a C^4/Z_k orbifold. In particular, for $k = 1$ and 2, ABJM conjectured that their theory describes the N M2-branes respectively in the flat space and on a R^8/Z_2 orbifold, and then might have $\mathcal{N} = 8$ supersymmetry. For $N = 2$, this theory has extra symmetries and is the same as the BLG theory [44].

On the other hand, D-branes are the hypersurfaces on which the open strings can end, and their dynamics is described by open string field theory [89]. The low-energy world-volume action for D-branes can be obtained by calculating the string scattering amplitudes [90]

or by using the T-duality [91]. As usual in string theory, there are high-order $\alpha' = \ell_s^2$ corrections, where ℓ_s is the string length scale. For a single D-brane, the D-brane action, which includes all order corrections in the gauge field strength but not its derivatives, takes the Dirac-Born-Infeld (DBI) form [92]. For multiple coincident D-branes, Tseytlin assumed that all the commutator terms should be treated as covariant derivative terms for gauge field strength, and thus should not be included in the effective action [90]. And he proposed that the action is the symmetrized trace of the direct non-Abelian generalization of the DBI action [90]. This non-Abelian DBI action gives the correct terms up to the order F^4 that were completely determined previously [93, 94]. But it fails for the higher order terms [95, 96]. Because the F^3 terms can always be written as the commutator terms, they are not interesting in the discussions of the D-brane effective action.

With the multiple M2-brane and D2-brane theories, we can study the deep relation between them. As we know, the full effective action of a D2-brane can be obtained by the reduction of the eleven-dimensional supermembrane action [97]. So, whether we can obtain the effective non-Abelian action for multiple D2-branes from the reduction of the BLG and ABJM theories is an interesting open question. Mukhi and Papageorgakis proposed a novel Higgs mechanism by giving vacuum expectation value (VEV) to a scalar field, which can promote the topological Chern-Simons gauge fields to dynamical gauge fields [8]. And they indeed obtained the maximally supersymmetric Yang-Mills theory for two D2-branes from the BLG theory at the leading order. Also, there exists a series of high-order corrections [8].

In this paper, we consider the derivation of the multiple D2-brane effective action from the multiple M2-branes in the BLG and ABJM theories in details. Concentrating on pure Yang-Mills fields, we show that the high-order F^3 and F^4 terms are commutator terms, and argue that all the high-order terms are also commutator terms. Thus, these high-order terms are irrelevant to the multiple D2-brane effective action. Note that the (gauged) BF membrane theory does not have high-order terms, the BLG theory, the (gauged) BF membrane theory, and the ABJM theory give the same D2-brane effective action. In order to generate the non-trivial high-order F^4 terms, inspired by the derivation of a single D2-brane from a M2-brane [97], we consider the curved M2-branes and introduce an auxiliary field. In particular, the VEV of the scalar field in the novel Higgs mechanism depends on the auxiliary field. After we integrate out the massive gauge fields and auxiliary field, we indeed obtain the high-order F^4 terms in the D2-brane effective action from the BLG theory and the ABJM theory with $SU(2) \times SU(2)$ gauge group. However, we still can not obtain the correct F^4 terms in the generic ABJM theories with gauge groups $U(N) \times U(N)$ and $SU(N) \times SU(N)$ for $N > 2$. The reason might be that the $SU(2) \times SU(2)$ gauge theory has three-dimensional $\mathcal{N} = 8$ superconformal symmetry while the $U(N) \times U(N)$ and $SU(N) \times SU(N)$ gauge theories with $N > 2$ may only have three-dimensional $\mathcal{N} = 6$ superconformal symmetry [72]. We also

briefly comment on the (gauged) BF membrane theory.

This paper is organized as follows. In Section II, we briefly review the novel Higgs mechanism in the BLG theory and (gauged) BF membrane theory, and study the novel Higgs mechanism in the ABJM theory. In Section III, we calculate the effective D2-brane action with the leading order F^2 , and high-order F^3 and F^4 terms from M2-branes. In Section IV, we generate the high-order F^4 terms by considering the curved M2-branes and introducing an auxiliary field. Our discussion and conclusions are given in Section V.

II. NOVEL HIGGS MECHANISM

In this Section, we briefly review the novel Higgs mechanism from M2-branes to D2-branes in the BLG theory and (gauged) BF membrane theory, and study it in the ABJM theory.

A. The BLG Theory and BF Membrane Theory

In the Lagrangian for the BLG theory with gauge group $SO(4)$ [4], we define

$$f^{abcd} \equiv f\epsilon^{abcd}, \quad f = \frac{2\pi}{k}, \quad (1)$$

where k is the level of the Chern-Simons terms. We also make the following transformation on the Yang-Mills fields

$$A_{\mu AB} \longrightarrow \frac{1}{f} A_{\mu AB}. \quad (2)$$

Then the Lagrangian for the BLG theory with gauge group $SO(4)$ becomes

$$\begin{aligned} \mathcal{L} = & -\frac{1}{2} D^\mu X^{AI} D_\mu X_A^I + \frac{i}{2} \bar{\Psi}^A \Gamma^\mu D_\mu \Psi_A + \frac{if}{4} \bar{\Psi}_B \Gamma_{IJ} X_C^I X_D^J \Psi_A \epsilon^{ABCD} \\ & -V(X) + \frac{1}{2f} \epsilon^{\mu\nu\lambda} (\epsilon^{ABCD} A_{\mu AB} \partial_\nu A_{\lambda CD} + \frac{2}{3} \epsilon^{CDA}{}_G \epsilon^{EFG B} A_{\mu AB} A_{\nu CD} A_{\lambda EF}), \end{aligned} \quad (3)$$

where $A = 1, 2, 3, 4$, $I = 1, 2, \dots, 8$, and

$$V(X) = \frac{f^2}{12} \epsilon_{ABCD} \epsilon_{EFG}{}^D X^{A(I)} X^{B(J)} X^{C(K)} X^{E(I)} X^{F(J)} X^{G(K)}. \quad (4)$$

As we know, the strong coupling limit of Type IIA theory is M-theory, and the coupling constant in Type IIA theory is related to the radius of the circle of the eleventh dimension in M-theory. Thus, for D2-branes, the gauge coupling constant is also related to the radius of the circle of the eleventh dimension. And at the strong coupling limit the D2-branes become M2-branes. To derive the D2-branes from M2-branes via the novel Higgs mechanism, we

compactify the M-theory on the circle of the eleventh dimension by giving VEV to a linear combination of the scalar fields $X^{A(I)}$ [8]. Because we have the $SO(8)$ R-symmetry and $SO(4)$ gauge symmetry, we can always make the rotation so that only the component $\langle X^{8(\phi)} \rangle$ develops a VEV

$$\langle X^{8(\phi)} \rangle = v_0 = \frac{v}{\sqrt{f}}, \quad (5)$$

where we split the index A into two sets $a = 1, 2, 3$ and $\phi = 4$. In addition, the gauge fields are splitted into A_μ^a and B_μ^a

$$A_\mu^a \equiv A_\mu^{a\phi}, \quad B_\mu^a \equiv \frac{1}{2} \epsilon^a{}_{bc} A_\mu^{bc}. \quad (6)$$

And then the Chern-Simons terms can be rewritten as

$$\frac{1}{2} \epsilon^{\mu\nu\lambda} \epsilon^{ABCD} A_{\mu AB} \partial_\nu A_{\lambda CD} = 4 \epsilon^{\mu\nu\lambda} B_\mu^a \partial_\nu A_{\lambda a}, \quad (7)$$

$$\frac{1}{3} \epsilon^{\mu\nu\lambda} \epsilon_{CDA}{}^G \epsilon_{EFG B} A_\mu^{AB} A_\nu^{CD} A_\lambda^{EF} = -4 \epsilon^{\mu\nu\lambda} \epsilon_{abc} B_\mu^a A_\nu^b A_\lambda^c - \frac{4}{3} \epsilon^{\mu\nu\lambda} \epsilon_{abc} B_\mu^a B_\nu^b B_\lambda^c, \quad (8)$$

where we neglect the total derivative term. Combining these two terms, the Chern-Simons action becomes

$$\mathcal{L}_{\text{CS}} = \frac{1}{f} \left(2 \epsilon^{\mu\nu\lambda} B_\mu^a F_{\nu\lambda a} - \frac{4}{3} \epsilon^{\mu\nu\lambda} \epsilon_{abc} B_\mu^a B_\nu^b B_\lambda^c \right), \quad (9)$$

where $F_{\nu\lambda a} = \partial_\nu A_{\lambda a} - \partial_\lambda A_{\nu a} - 2 \epsilon_{abc} A_\nu^b A_\lambda^c$ is the field strength for the gauge field A_μ^a . Similarly, the kinetic terms for the scalar fields are

$$D_\mu X^{a(I)} = \partial_\mu X^{a(I)} + \epsilon^a{}_{BCD} A_\mu^{CD} X^{B(I)} = \partial_\mu X^{a(I)} - 2 \epsilon^a{}_{cb} A_\mu^c X^{b(I)} + 2 B_\mu^a X^{\phi(I)}, \quad (10)$$

$$D_\mu X^{\phi(I)} = \partial_\mu X^{\phi(I)} - 2 B_{\mu a} X^{a(I)}. \quad (11)$$

Substituting these back into the action and setting $X^{\phi(8)} \rightarrow X^{\phi(8)} + v$, we obtain the terms involving B_μ^a from the scalar kinetic terms

$$\begin{aligned} \mathcal{L} = & -\frac{2v^2}{f} B_\mu^a B_a^\mu - \frac{4v}{\sqrt{f}} X^{8\phi} B_\mu^a B_a^\mu - 2 X^{8\phi} X^{8\phi} B_\mu^a B_a^\mu - 2 B_\mu^a B_a^\mu X^{\phi(i)} X_{\phi(i)} \\ & - 2 B_\mu^a X^{\phi(i)} D^\mu X_a^{(i)} - \frac{2v}{\sqrt{f}} B_\mu^a D^\mu X_a^{(8)} - 2 X^{8\phi} B_\mu^a D^\mu X_a^{(8)} \\ & - 2 B_{\mu a} X^{a(I)} B_b^\mu X^{b(I)} + 2 B_a^\mu X^{a(I)} \partial_\mu X^{\phi(I)}, \end{aligned} \quad (12)$$

where $i = 1, 2, \dots, 7$, and the new defined covariant derivative is $D_\mu X^{a(I)} = \partial_\mu X^{a(I)} - 2 \epsilon^a{}_{bc} A_\mu^b X^{c(I)}$. Therefore, the relevant Lagrangian for pure Yang-Mills fields is

$$\mathcal{L}_{\text{YM}} = \frac{1}{f} \left(-2v^2 B_\mu^a B_a^\mu + 2 \epsilon^{\mu\nu\lambda} B_\mu^a F_{\nu\lambda a} - \frac{4}{3} \epsilon^{\mu\nu\lambda} \epsilon_{abc} B_\mu^a B_\nu^b B_\lambda^c \right). \quad (13)$$

Next, we would like to briefly review the result of the novel Higgs mechanism in the BF membrane theory [24, 25, 26]. Here, we follow the convention in Ref. [25] except that we choose

$$(B_\mu)_a \equiv \frac{1}{2}(A_\mu)_{bc} f_a{}^{bc} . \quad (14)$$

In this theory, the equation of motion for ghost field X_-^I gives the constraint $\partial^2 X_+^I = 0$. So, we can give a constant VEV to X_+^8 , *i.e.*, $X_+^8 = v$. And then we obtain the relevant Lagrangian for pure gauge fields

$$\mathcal{L} = -2v^2 B_\mu^a B_a^\mu + 2\epsilon^{\mu\nu\lambda} B_\mu^a F_{\nu\lambda}^a . \quad (15)$$

It should be noted that unlike the Lagrangian in Eq. (13) in the BLG theory, there is no cubic term for B_μ^a in above Lagrangian. And this is one of the motivations of the work [47] which showed that the gauged BF membrane theory might be equivalent to the maximally supersymmetric three-dimensional Yang-Mills theory via a duality transformation due to de Wit, Nicolai and Samtleben [86].

After gauging the shift symmetries for the ghost fields X_-^I and Ψ_- in the BF membrane theory [40, 43] by introducing new gauge fields, we could make the gauge choice to decouple the ghost states. And the equation of motion for the new gauge fields gives the constraint $\partial_\mu X_+^I = 0$, which indicates that X_+^I must be a constant. We emphasize that in this case the relevant Lagrangian for pure Yang-Mills fields is still given by Eq. (15).

B. The ABJM Theory

Very recently, Aharony, Bergman, Jafferis and Maldacena (ABJM) have constructed three-dimensional $U(N) \times U(N)$ and $SU(N) \times SU(N)$ Chern-Simons gauge theories with $\mathcal{N} = 6$ superconformal symmetry. From the brane constructions, they argued that the $U(N) \times U(N)$ theory at Chern-Simons level k describes the low-energy limit of N M2-branes probing a C^4/Z_k singularity. It was conjectured that for $k = 1$ and 2, the ABJM theory respectively describes N M2-branes in flat space and on a R^8/Z_2 orbifold, and then may have $\mathcal{N} = 8$ supersymmetry. For $N = 2$, this theory has additional symmetries and becomes identical to the BLG theory. In this subsection, we will study the novel Higgs mechanism in the ABJM theory.

Following the convention in Ref. [46], we can write the explicit Lagrangian in ABJM theory as follows

$$\begin{aligned} \mathcal{L} = & 2K\epsilon^{\mu\nu\lambda}\text{Tr} \left(A'_\mu \partial_\nu A'_\lambda + \frac{2i}{3} A'_\mu A'_\nu A'_\lambda - \hat{A}_\mu \partial_\nu \hat{A}_\lambda - \frac{2i}{3} \hat{A}_\mu \hat{A}_\nu \hat{A}_\lambda \right) \\ & - \text{Tr} \left((\mathcal{D}_\mu Z)^\dagger \mathcal{D}^\mu Z + (\mathcal{D}_\mu W)^\dagger \mathcal{D}^\mu W - i\zeta^\dagger \gamma^\mu \mathcal{D}_\mu \zeta - i\omega^\dagger \gamma^\mu \mathcal{D}_\mu \omega \right) \\ & - V_{\text{ferm}} - V_{\text{bos}} , \end{aligned} \quad (16)$$

where

$$K = \frac{k}{8\pi} , \quad (17)$$

$$Z^1 = X^1 + iX^5 , \quad Z^2 = X^2 + iX^6 , \quad W_1 = X^{3\dagger} + iX^{7\dagger} , \quad W_2 = X^{4\dagger} + iX^{8\dagger} , \quad (18)$$

where X^i belongs to the bifundamental representation of $U(N) \times U(N)$ or $SU(N) \times SU(N)$, and here we do not present the potential V_{ferm} and V_{bos} since they are irrelevant in the following discussions. For our convention, we choose

$$\text{Tr}(T^a T^b) = \frac{1}{2} \delta_{ab} , \quad [T^a, T^b] = i f_{abc} T^c , \quad (19)$$

where $T^{a,b,c}$ are the generators of the corresponding gauge group.

Similar to the novel Higgs mechanism in the BLG theory, we give the diagonal VEV to X^8 as follows

$$\langle X^8 \rangle = v_0 I_{N \times N} = v \sqrt{K} I_{N \times N} , \quad (20)$$

where $I_{N \times N}$ is the N by N identity matrix. Also, we define

$$A_\mu = \frac{1}{2} (A'_\mu + \hat{A}_\mu) , \quad B_\mu = \frac{1}{2} (A'_\mu - \hat{A}_\mu) . \quad (21)$$

So we have

$$A'_\mu = A_\mu + B_\mu , \quad \hat{A}_\mu = A_\mu - B_\mu . \quad (22)$$

From the kinetic term for W_2 and the Chern-Simons terms, we obtain the relevant Lagrangian for pure Yang-Mills fields

$$\mathcal{L}_{\text{YM}} = K \left(-2v^2 B_\mu^a B_a^\mu + 2\epsilon^{\mu\nu\lambda} B_\mu^a F_{\nu\lambda} - \frac{2}{3} \epsilon^{\mu\nu\lambda} f_{abc} B_\mu^a B_\nu^b B_\lambda^c \right) , \quad (23)$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + i[A_\mu, A_\nu]$. Note that the BLG theory with $SO(4)$ gauge group is the same as the ABJM theory with $SU(2) \times SU(2)$ gauge group, so we can obtain the Lagrangian in Eq. (13) from that in the above Eq. (23) by rescaling f_{abc} .

III. EFFECTIVE ACTION FOR THE PURE GAUGE FIELDS

Because B_μ^a is massive, we will calculate the effective action for pure Yang-Mills fields by integrating it out. Due to the absence of the cubic term for B_μ^a in the (gauged) BF membrane theory, we do not have the high-order corrections in the effective action of gauge fields. Thus, we will concentrate on the BLG theory and ABJM theory. The relevant Lagrangians for pure gauge fields are the same for the BLG theory and the ABJM theory

with $SU(2) \times SU(2)$ gauge symmetry, and the ABJM theory is more general. Thus, we will use the Lagrangian in Eq. (23) in the following discussions.

From the Lagrangian in Eq. (23), we get the equation of motion for B_μ^a

$$B_a^\mu = \frac{1}{2v^2} \epsilon^{\mu\nu\lambda} F_{\nu\lambda} - \frac{1}{2v^2} \epsilon^{\mu\nu\lambda} f_{abc} B_\nu^b B_\lambda^c . \quad (24)$$

We can solve the above equation by parametrizing the solution in $1/v^2$ expansion

$$B_a^\mu = \sum_n \frac{1}{v^{2n}} (C_{2n})_a^\mu . \quad (25)$$

Substituting it back into Eq. (24), we obtain

$$\sum_n \frac{1}{v^{2n}} (C_{2n})_a^\mu = \frac{1}{2v^2} \epsilon^{\mu\nu\lambda} F_{\nu\lambda} - \frac{1}{2v^2} \sum_{n,m} \frac{1}{v^{2n+2m}} \epsilon^{\mu\nu\lambda} f_{abc} (C_{2n})_\nu^b (C_{2m})_\lambda^c + \dots . \quad (26)$$

Because we only know for sure the high-order terms up to the order of F^4 in D2-brane effective action [90, 95, 96], we only need to calculate the solution to Eq. (24) up to the order of $1/v^{10}$ or $(C_{10})_a^\mu$. And the non-vanishing terms in the solution are

$$(C_2)_a^\mu = \frac{1}{2} \epsilon^{\mu\nu\lambda} F_{\nu\lambda} , \quad (C_6)_a^\mu = -\frac{1}{2} \epsilon^{\mu\nu\lambda} f_{abc} (C_2)_\nu^b (C_2)_\lambda^c , \quad (27)$$

$$(C_{10})_a^\mu = -\epsilon^{\mu\nu\lambda} f_{abc} (C_2)_\nu^b (C_6)_\lambda^c . \quad (28)$$

Integrating B_μ^a out, we get the Lagrangian for pure Yang-Mills fields

$$\mathcal{L}_{\text{YM}} = \mathcal{L}_{\text{YM}}^{(2)} + \mathcal{L}_{\text{YM}}^{(3)} + \mathcal{L}_{\text{YM}}^{(4)} + \dots , \quad (29)$$

where

$$\mathcal{L}_{\text{YM}}^{(2)} = \frac{2K}{v^2} (C_2)_\mu^a (C_2)_a^\mu , \quad (30)$$

$$\mathcal{L}_{\text{YM}}^{(3)} = -\frac{2K}{3v^6} \epsilon^{\mu\nu\lambda} f_{abc} (C_2)_\mu^a (C_2)_\nu^b (C_2)_\lambda^c , \quad (31)$$

$$\mathcal{L}_{\text{YM}}^{(4)} = -\frac{2K}{v^{10}} (C_6)_\mu^a (C_6)_a^\mu - \frac{2K}{v^{10}} \epsilon^{\mu\nu\lambda} f_{abc} (C_2)_\mu^a (C_2)_\nu^b (C_6)_\lambda^c . \quad (32)$$

Using Eqs. (27) and (28), and the useful identities in the Appendix A, we obtain

$$\mathcal{L}_{\text{YM}}^{(2)} = -\frac{2K}{v^2} \text{Tr} (F^2) , \quad (33)$$

$$\mathcal{L}_{\text{YM}}^{(3)} = \frac{i4K}{3v^6} \text{Tr} (F_{\alpha_1\beta_1} [F^{\beta_1\beta_3}, F_{\beta_3}^{\alpha_1}]) , \quad (34)$$

$$\mathcal{L}_{\text{YM}}^{(4)} = \frac{K}{2v^{10}} \text{Tr} \left([F^{\rho\sigma}, F^{\eta\delta}] [F_{\eta\delta}, F_{\rho\sigma}] \right) . \quad (35)$$

Thus, $\mathcal{L}_{\text{YM}}^{(2)}$ is the kinetic term for the gauge fields A_a^μ and is the leading order of the supersymmetric Yang-Mills effective action. Moreover, the gauge coupling in the BLG theory is

$$g_{YM}^2 = \frac{fv^2}{4} = \frac{f^2 v_0^2}{4} \propto \frac{v_0^2}{k^2} , \quad (36)$$

and the gauge coupling in the ABJM theory is

$$g_{YM}^2 = \frac{v^2}{4K} = \frac{v_0^2}{4K^2} \propto \frac{v_0^2}{k^2} . \quad (37)$$

So for very large v_0 and k , we can still keep the gauge coupling as a fixed constant. For D2-branes, the gauge coupling is related to the string coupling and the string length as follows

$$g_{YM} = \left(\frac{g_s}{\ell_s} \right)^{\frac{1}{2}} . \quad (38)$$

And then for the fixed string coupling, we have $g_{YM}^2 \propto \alpha^{-1/2}$. Therefore, $1/v$ is proportional to $\alpha^{1/4}$, $\mathcal{L}_{\text{YM}}^{(3)}$ and $\mathcal{L}_{\text{YM}}^{(4)}$ are proportional to $g_{YM}^{-2} \alpha'$ and $g_{YM}^{-2} \alpha'^2$, respectively. In short, they are at the correct orders according to the α' expansion.

Because $\mathcal{L}_{\text{YM}}^{(3)}$ and $\mathcal{L}_{\text{YM}}^{(4)}$ only have commutator terms, these high-order terms are covariant derivative terms and then do not contribute to the effective action for the D2-branes [90]. We conjecture that all the high-order terms obtained by this approach are the commutator terms. The point is that the equation of motion for B_μ^a in Eq. (24) can be rewritten as follows

$$B^{a\mu} = \frac{1}{2v^2} \epsilon^{\mu\nu\lambda} F_{\nu\lambda}^a + \frac{i}{v^2} \epsilon^{\mu\nu\lambda} \text{Tr} (T^a [B_\nu, B_\lambda]) . \quad (39)$$

Because all the high-order terms originally come from the last term in the above equation which is a commutator term, all the high-order terms should be the commutator terms and then the covariant terms. Thus, modulating the commutator terms or covariant derivative terms, we only have the kinetic term for the gauge fields A_a^μ from the BLG and ABJM theories, which is the leading order in the D2-brane effective action. And then the effective action for pure Yang-Mills fields from the BLG and ABJM theories is the same as that from the (gauged) BF membrane theory after we integrate B_a^μ out. Therefore, how to obtain the non-trivial F^4 terms in the D2-brane effective action from the BLG theory, the (gauged) BF membrane theory, and the ABJM theory is still a big problem.

IV. D2-BRANES FROM THE CURVED M2-BRANES

In spired by the derivation of a single D2-brane from a M2-brane [97], we would like to consider the multiple curved M2-branes. To employ the trick in Ref. [97], we only need to introduce gravity. For simplicity, we do not consider the dilaton, the vector and scalar fields in the eleven-dimensional metric due to compactification, and RR fields, etc. And our ansatz for the Lagrangian of the curved M2-branes is

$$\mathcal{L}_{\text{Curved}} = -\beta_0 \sqrt{-\det(g)} + \sqrt{-\det(g)} \mathcal{L}_{\text{M2s}} , \quad (40)$$

where β_0 is a positive constant like membrane tension, $g_{\mu\nu}$ is the induced metric on the world-volume of multiple M2-branes, and \mathcal{L}_{M2s} is formally given in Eq. (3) for the BLG theory or in Eq. (16) for the ABJM theory. In \mathcal{L}_{M2s} , we need to replace $\eta_{\mu\nu}$ and ∂_α by $g_{\mu\nu}$ and ∇_α , respectively. Also, we replace $\epsilon_{\mu\nu\lambda}$ by $\varepsilon_{\mu\nu\lambda} = \sqrt{-g}\epsilon_{\mu\nu\lambda}$ which will be covariant under coordinate transformation. This is a natural action for the multiple M2-branes in the curved space-time since it can come back to flat theory after we decouple the gravity.

Similar to the discussions in Ref. [97], we introduce an auxiliary filed u and rewrite the above Lagrangian as follows

$$\mathcal{L}_{\text{Curved}} = \frac{\beta_0^2}{2u} \det(g) - \frac{u}{2} + \sqrt{-\det(g)} \mathcal{L}_{\text{M2s}} . \quad (41)$$

We can obtain the Lagrangian in Eq. (40) from Eq. (41) by integrating out the auxiliary filed u .

To match the convention in [90], we give the following VEV to the scalar field ϕ

$$\langle \phi \rangle = \left(\frac{8u}{\sqrt{-\det(g)}} \right)^{1/2} \frac{K'}{\beta_0} \frac{1}{2\pi\alpha'} I_{N \times N} , \quad (42)$$

where we can take $\phi = X^{8(\phi)}$, $K' = 1/f$, and $N = 1$ in the BLG theory, take $\phi = X_+^8$, $K' = 1$ and $N = 1$ in the (gauged) BF membrane theory, and take $\phi = X^8$ and $K' = K$ in the ABJM theory. Thus, the relevant Lagrangian is

$$\begin{aligned} \mathcal{L}_{\text{Curved}} = & \frac{\beta_0^2}{2u} \det(g) - \frac{u}{2} + \sqrt{-\det(g)} \left(-2\langle \phi^2 \rangle B_\mu^a B_a^\mu + 2K' \varepsilon^{\mu\nu\lambda} B_\mu^a F_{\nu\lambda}^a \right. \\ & \left. - \frac{2}{3} K' \varepsilon^{\mu\nu\lambda} f_{abc} B_\mu^a B_\nu^b B_\lambda^c \right) . \end{aligned} \quad (43)$$

Using the results of the novel Higgs mechanism in the Section III and neglecting the commutator terms for the A_μ^a field strength, we obtain

$$\mathcal{L} = \frac{\beta_0^2}{2u} \det(g) \left(1 + \frac{(2\pi\alpha')^2}{4} F^{a\mu\nu} F_{\mu\nu}^a \right) - \frac{u}{2} . \quad (44)$$

Moreover, we use the following identity for the 3×3 matrices that is proved in the Appendix A

$$\text{Str det } (g + 2\pi\alpha' F) = \det(g) \left(1 + \frac{(2\pi\alpha')^2}{4} F^{a\mu\nu} F_{\mu\nu}^a \right), \quad (45)$$

where “Str” is the symmetrized trace that acts on the gauge group indices, and “det” acts on the world-volume coordinate indices. Integrating out the auxiliary field u , we obtain the Lagrangian for multiple D2-brane effective action

$$\mathcal{L} = -\beta_0 \sqrt{-\text{Str det } (g + 2\pi\alpha' F)}. \quad (46)$$

However, the well-known Lagrangian for the multiple D2-brane DBI action is [90]

$$\mathcal{L} = -c_0 \text{Str} \left[\sqrt{-\det (g + 2\pi\alpha' F)} \right], \quad (47)$$

where c_0 is a constant. Because in general the Lagrangian in Eq. (46) is not equivalent to that in Eq. (47), we still can not get the correct F^4 terms for generic case.

Interestingly, for gauge symmetry $SU(2) \times SU(2)$ in the BLG theory, or the (gauged) BF membrane theory, or the ABJM theory, we indeed can get the correct F^4 terms. Let us prove it in the following. From the Lagrangian in Eq. (46), we obtain

$$\mathcal{L} = -\beta_0 \sqrt{-\det(g) \left(1 + \frac{(2\pi\alpha')^2}{4} F^{a\mu\nu} F_{\mu\nu}^a \right)}. \quad (48)$$

Expanding the above Lagrangian, we have the relevant Lagrangian for pure Yang-Mills fields at the Minkowski space-time limit

$$\mathcal{L} = -\frac{\beta_0(2\pi\alpha')^2}{4} \text{Tr} \left[F^{\mu\nu} F_{\mu\nu} - \frac{(2\pi\alpha')^2}{4} (F^{\mu\nu} F_{\mu\nu})^2 + \dots \right]. \quad (49)$$

From the known effective action for multiple D2-branes, the relevant Lagrangian for pure Yang-Mills fields up to the F^4 terms is [90]

$$\begin{aligned} \mathcal{L}_{DBI} = c_1 \text{Tr} \left\{ F^{\mu\nu} F_{\mu\nu} - \frac{1}{3} (2\pi\alpha')^2 \left(F^{\mu\nu} F_{\rho\nu} F_{\mu\sigma} F^{\rho\sigma} + \frac{1}{2} F^{\mu\nu} F_{\rho\nu} F^{\rho\sigma} F_{\mu\sigma} \right. \right. \\ \left. \left. - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} F^{\rho\sigma} F_{\rho\sigma} - \frac{1}{8} F^{\mu\nu} F^{\rho\sigma} F_{\mu\nu} F_{\rho\sigma} \right) + \dots \right\}, \end{aligned} \quad (50)$$

where $c_1 = \pi^2 \alpha'^2 c_0$. For gauge group $SU(2)$, we obtain

$$\mathcal{L}_{DBI} = c_1 \text{Tr} \left\{ F^{\mu\nu} F_{\mu\nu} - (2\pi\alpha')^2 \left(\frac{1}{8} (F^{\mu\nu} F_{\mu\nu})^2 + \frac{1}{24} F^{\mu\nu} F^{\rho\sigma} [F_{\mu\nu}, F_{\rho\sigma}] \right) + \dots \right\}. \quad (51)$$

Therefore, neglecting the commutator terms and rescaling the gauge fields, we can show that the correct F^4 terms in the effective D2-brane action in Eq. (49) from the two M2-branes in the BLG and ABJM theories are equivalent to these in the known DBI action in Eq. (51).

In short, we can generate the correct F^4 terms in the effective D2-brane action from the BLG theory and the ABJM theory with gauge group $SU(2) \times SU(2)$. However, we can not get the correct F^4 terms from the ABJM theory with $U(N) \times U(N)$ and $SU(N) \times SU(N)$ gauge symmetries for $N > 2$. It seems to us that the reasons are the following: the BLG theory and the ABJM theory with gauge group $SU(2) \times SU(2)$ have three-dimensional $\mathcal{N} = 8$ superconformal symmetry while the ABJM theory with $U(N) \times U(N)$ and $SU(N) \times SU(N)$ gauge symmetries for $N > 2$ might only have three-dimensional $\mathcal{N} = 6$ superconformal symmetry [72]. However, for the (gauged) BF membrane theory, although the constraint $\nabla_\mu X_+^8 = 0$ is still satisfied, it might be equivalent to three-dimensional $\mathcal{N} = 8$ supersymmetric Yang-Mills theory. In particular, for the (gauged) BF membrane theory with $SU(2) \times SU(2)$ gauge symmetry, we can generate the correct F^4 terms since it is similar to the corresponding BLG and ABJM theories.

V. DISCUSSION AND CONCLUSIONS

Using the novel Higgs mechanism, we considered the derivation of the multiple D2-brane effective action for pure Yang-Mills fields from the multiple M2-branes in the BLG theory and the ABJM theory. We showed that the high-order F^3 and F^4 terms are commutator terms, and we argued that all the high-order terms are commutator terms as well. Thus, these high-order terms do not contribute to the multiple D2-brane effective action. In order to generate the non-trivial high-order F^4 terms and inspired by the derivation of one D2-brane from one M2-brane, we considered the curved M2-branes and introduce an auxiliary field. In particular, the VEV of the scalar field in the novel Higgs mechanism depends on the auxiliary field. After we integrate out the massive gauge fields and auxiliary field, we obtain the correct high-order F^4 terms in the D2-brane effective action from the BLG theory and the ABJM theory with $SU(2) \times SU(2)$ gauge group. However, we still can not obtain the correct F^4 terms in the generic ABJM theory with gauge groups $U(N) \times U(N)$ and $SU(N) \times SU(N)$ for $N > 2$. This might be related to the possible fact that the $SU(2) \times SU(2)$ gauge theory has three-dimensional $\mathcal{N} = 8$ superconformal symmetry while the $U(N) \times U(N)$ and $SU(N) \times SU(N)$ gauge theories for $N > 2$ might only have three-dimensional $\mathcal{N} = 6$ superconformal symmetry. We also briefly comment on the (gauged) BF membrane theory.

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APPENDIX A: MATHEMATICAL IDENTITIES

In this appendix we collect or prove the useful identities in this paper:

(1) Two useful identities about ϵ in three-dimensional Minkowski space-time

$$\epsilon^{\mu\nu\lambda}\epsilon_\lambda{}^{\rho\sigma} = (-\eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho}) , \quad (\text{A1})$$

$$\epsilon^{\mu\gamma\lambda}\epsilon^{\nu\rho\sigma} = -\eta^{\mu\nu}(\eta^{\gamma\rho}\eta^{\lambda\sigma} - \eta^{\gamma\sigma}\eta^{\lambda\rho}) + \eta^{\mu\rho}(\eta^{\gamma\nu}\eta^{\lambda\sigma} - \eta^{\gamma\sigma}\eta^{\lambda\nu}) - \eta^{\mu\sigma}(\eta^{\gamma\nu}\eta^{\lambda\rho} - \eta^{\gamma\rho}\eta^{\lambda\nu}) . \quad (\text{A2})$$

(2) Let us prove the identity in Eq. (45) which is right for the Abelian and non-Abelian cases

$$\begin{aligned} \text{Str det}(g + aF) &= \text{det}(g_{\mu\nu}) \text{Str det}(g^\nu{}_\lambda + aF^\nu{}_\lambda) \\ &= (\text{det } g) \text{Str } \epsilon_{\alpha_1\alpha_2\alpha_3}(g^1{}_{\alpha_1} + aF^1{}_{\alpha_1})(g^2{}_{\alpha_2} + aF^2{}_{\alpha_2})(g^3{}_{\alpha_3} + aF^3{}_{\alpha_3}) \\ &= (\text{det } g) \text{Str} \left[1 + aF^\alpha{}_\alpha + a^2 \left(\epsilon_{1\alpha_2\alpha_3} F^2{}_{\alpha_2} F^3{}_{\alpha_3} + \epsilon_{\alpha_1 2\alpha_3} F^1{}_{\alpha_1} F^3{}_{\alpha_3} \right. \right. \\ &\quad \left. \left. + \epsilon_{\alpha_1\alpha_2 3} F^1{}_{\alpha_1} F^2{}_{\alpha_2} \right) + a^3 \epsilon_{\alpha_1\alpha_2\alpha_3} F^1{}_{\alpha_1} F^2{}_{\alpha_2} F^3{}_{\alpha_3} \right] \\ &= (\text{det } g) \left(1 + \frac{a^2}{4} F^{a\mu\nu} F_{\mu\nu}^a \right) , \end{aligned} \quad (\text{A3})$$

where $a = 2\pi\alpha'$.

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